T.P.C. REVIEW MENU:
Degree and Radian Measurements

Assumption: Everyone learns differently so students should not always have to work on the same material at the same time.

Complete: All Entrée Items, 3 Appetizers OR desserts

**COMPILE IN ORDER OF LEARNING GOALS. STAPLE AND PUT INTO PORTFOLIO.**

| Learning Goal	Appetizer (Practice)	Entrée (Required)	Dessert (Above & Beyond)
1. Identify angle measurements in degrees (positive and negative)	☐ A1. Practice with Negative Angles	☐ E1. Identify Angles in Degrees	
2. Identify angle measurements in radians (positive and negative)		☐ E2. Identify Angles in Radian	☐ D2. Exercise 71-78 (p.369)
3. Calculate co-terminal angles in radian and degrees	☐ A3. Exercise 33-34 (p.368)	☐ E3. Co-terminal Angles	☐ D3. Rotations ☐ D3.1 Teach a peer _____ (signature)
4. Calculate supplementary and complementary angles in radian and degrees	☐ A4.1 Adding Fractions Practice ☐ A4.2 Fractions Workshop with Ms. Song _____ (initials)	☐ E4. Complementary and Supplementary Angles	☐ D4. Essay Question: Explain why $\frac{5\pi}{4}$ radian has neither complement or supplement. USE calculations! ☐ D4.1 Teach a peer _____ (signature)
5. Convert between radian and degrees.	☐ A5. Read Conversions between Degrees (p.364) + Exercise #47-50, 55-57	☐ E5. Converting Radians and Degrees	☐ D5. Essay Question: What is larger: one degree or one radian? Justify your answer using MULTIPLE explanations.

A1 A1. Practice with Negative Angles   Date_____ Period____
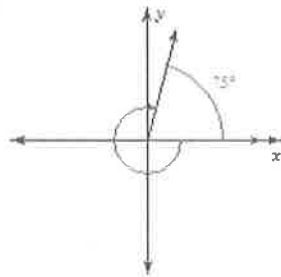
Find the measure of each angle.

1)
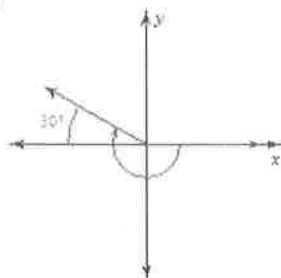
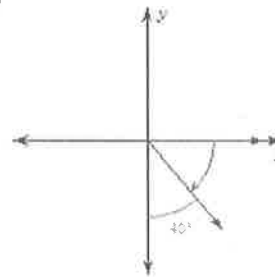2)
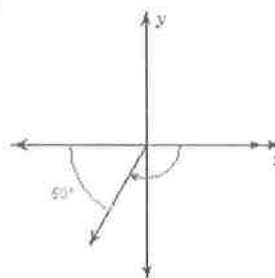
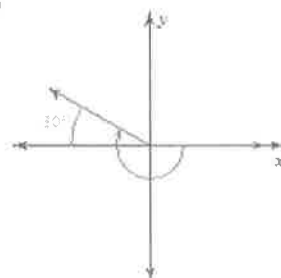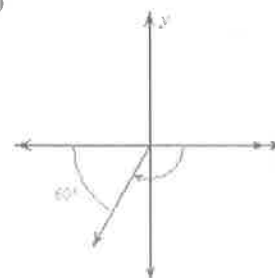

3)

4)

P1. Practice with Negative Angles   Date_____ Period____
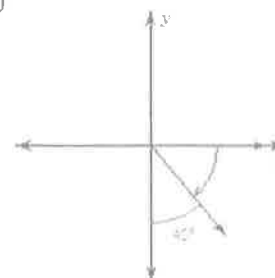
Find the measure of each angle.

1)

2)

3)

4)

Source: Esther Song, Niles West High School
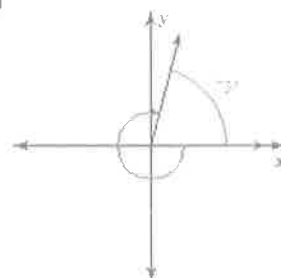
Answers to P1. Practice with Negative Angles (ID: 1)

1) −210°   2) −120°   3) −285°   4) −50°

Source: Esther Song, Niles West High School
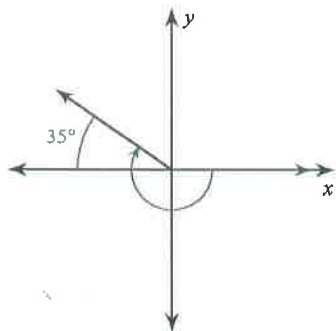
Algebra 2   Name_____

E1 Identify angles in degrees
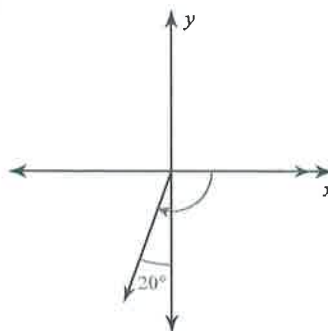
Date_____ Period_____

Find the measure of each angle.

1)

35°

2)

20°

3)

140°

4)

160°
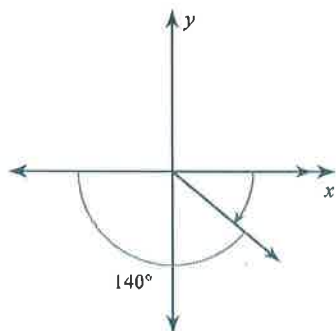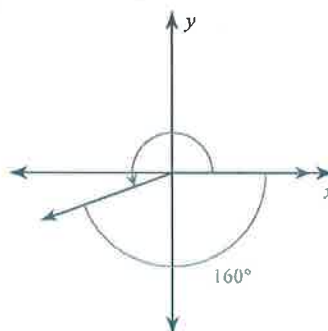
Draw an angle with the given measure in standard position.

5) 100°

6) 675°

7) −10°

8) −690°

Source: Esther Song, Niles West High School
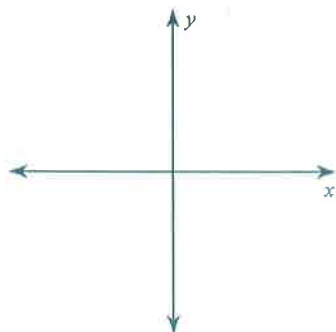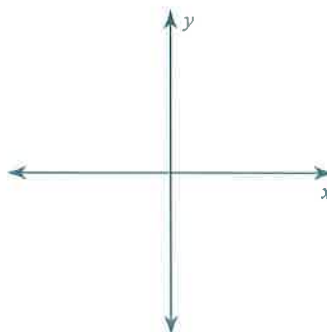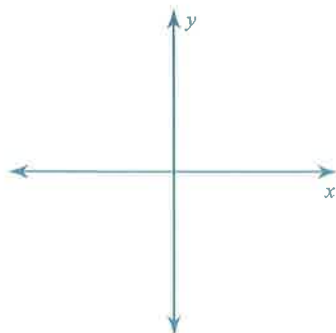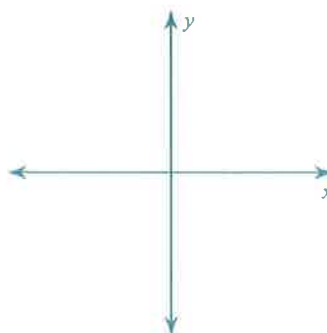
Answers to E1 Identify angles in degrees (ID: 1)

1) −215°

2) −110°

3) −40°

4) 200°

5)

6)

7)

8)

Name_____
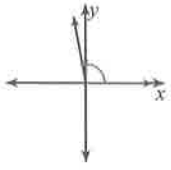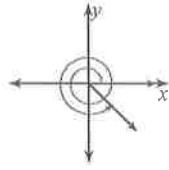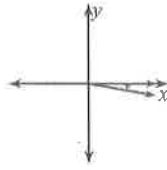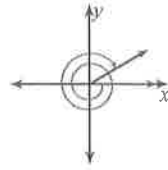
E2. Identify Angles in Radian

Date_____ Period_____

Draw an angle with the given measure in standard position.

1) $\dfrac{\pi}{3}$
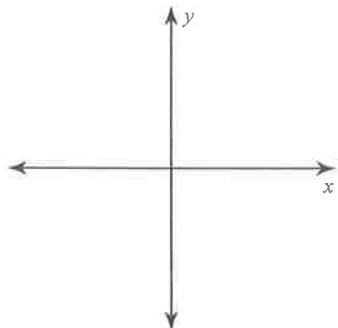
2) $-\dfrac{3\pi}{4}$
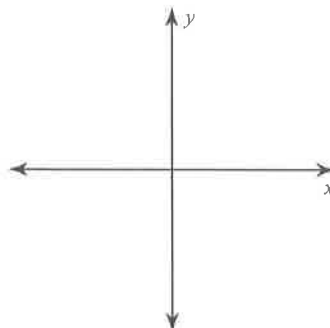
3) $\dfrac{5\pi}{4}$
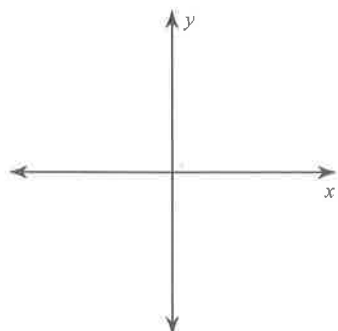
4) $\dfrac{11\pi}{6}$
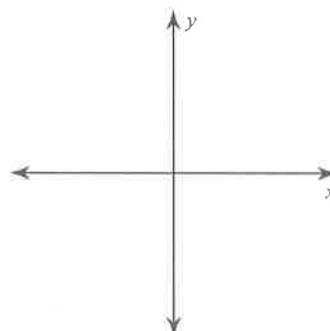
5) $-\dfrac{11\pi}{3}$
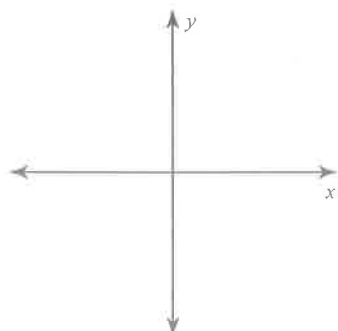
6) $-\dfrac{5\pi}{9}$
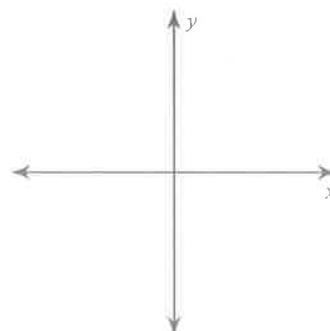
Answers to E2. Identify Angles in Radian (ID: 1)

1)

2)

3)

4)

5)

6)

D.& Exercise

SECTION 4.1 | Radian and Degree Measure   369

In Exercises 47–54, convert the measure from degrees to radians. Round to three decimal places.

47. 115°   **48.** 87.4°

49. −216.35°   **50.** −48.27°

51. 532°   **52.** 0.54°

53. −0.83°   **54.** 345°

In Exercises 55–62, convert the measure from radians to degrees. Round to three decimal places.

55. $\pi/7$ **56.** $5\pi/11$

57. $15\pi/8$ **58.** 6.5π

59. -4.2π **60.** 4.8

61. −2   **62.** −0.57

In Exercises 63–66, convert to decimal degree form.

63. (a) 54° 45′   (b) −128° 30′

64. (a) 245° 10′   (b) 2° 12′

65. (a) 85° 18′ 30″   (b) 330° 25″

66. (a) −135° 36″   (b) −408° 16′ 20″
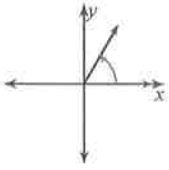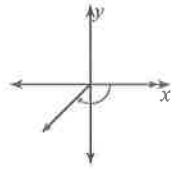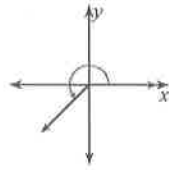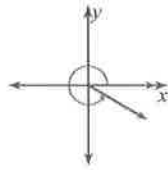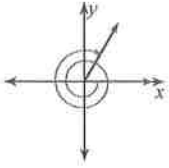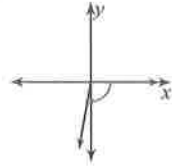
In Exercises 67–70, convert to D° M′ S″ form.

67. (a) 240.6°   (b) −145.8°

68. (a) −345.12°   (b) 0.45

69. (a) 2.5   (b) −3.58

70. (a) −0.355   (b) 0.7865

In Exercises 71–74, find the angle in radians.

In Exercises 75–78, find the radian measure of the central angle of a circle of the given radius that intercepts an arc of the given length.

	Radius	Arc Length
75.	15 inches	4 inches
76.	16 feet	10 feet
77.	14.5 centimeters	25 centimeters
78.	80 kilometers	160 kilometers

In Exercises 79–82, find the length of the arc on a circle of the given radius intercepted by the given central angle.

	Radius	Central Angle
79.	15 inches	180°
80.	9 feet	60°
81.	6 meters	2 radians
82.	40 centimeters	$3\pi/4$ radians

Distance Between Cities In Exercises 83–86, find the distance between the cities. Assume that earth is a sphere of radius 4000 miles and the cities are on the same meridian (one city is due north of the other).

	City	Latitude
83.	Dallas, Texas	32° 47′ 9″N
	Omaha, Nebraska	41° 15′ 42″N
84.	San Francisco, California	37° 46′ 39″N
	Seattle, Washington	47° 36′ 32″N
85.	Miami, Florida	25° 46′ 37″N
	Erie, Pennsylvania	42° 7′ 15″N
86.	Johannesburg, South Africa	26° 10′ S
	Jerusalem, Israel	31° 47′ N

87. ***Difference in Latitudes*** Assuming that earth is a sphere of radius 6378 kilometers, what is the difference in latitude of two cities, one of which is 600 kilometers due north of the other?

88. ***Difference in Latitudes*** Assuming that earth is a sphere of radius 6378 kilometers, what is the difference in latitude of two cities, one of which is 800 kilometers due north of the other?

Source: Esther Song, Niles West High School

A.3 #33-34

16. (a)

(b)

17. (a) $-9\pi/4$ (b) $-2\pi/15$

18. (a) $8\pi/9$ (b) $8\pi/45$

In Exercises 19–20, find (if possible) the complement and supplement of the angle.

19. (a) $\pi/3$ (b) $3\pi/4$

20. (a) 1   (b) 2

In Exercises 21–24, estimate the angle in degrees.

21.

22.

23.

24.
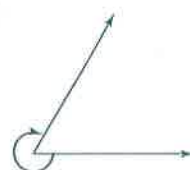
In Exercises 25–28, determine the quadrant in which the angle lies.

25. (a) $130°$ (b) $285°$

26. (a) $8.3°$ (b) $257° \, 30'$

27. (a) $-132° \, 50'$ (b) $-336°$

28. (a) $-260°$ (b) $-3.4°$

In Exercises 29–32, sketch the angle in standard position.

29. (a) $30°$ (b) $150°$

30. (a) $-270°$ (b) $-120°$

31. (a) $405°$ (b) $-480°$

32. (a) $750°$ (b) $-600°$

In Exercises 33–36, determine two coterminal angles (one positive and one negative) for the given angle. Give your answers in degrees.

33. (a)   (b)

34. (a)   (b)

35. (a) $300°$ (b) $740°$

36. (a) $-420°$ (b) $230°$

In Exercises 37 and 38, find (if possible) the complement and supplement of the angle.

37. (a) $18°$ (b) $115°$

38. (a) $79°$ (b) $150°$

In Exercises 39–42, express the angle in radian measure as a multiple of π. (Do not use a calculator.)

39. (a) $30°$ (b) $150°$

40. (a) $315°$ (b) $120°$

41. (a) $-20°$ (b) $-240°$

42. (a) $-270°$ (b) $144°$

In Exercises 43–46, express the angle in degree measure. (Do not use a calculator.)

43. (a) $3\pi/2$ (b) $7\pi/6$

44. (a) $-7\pi/12$ (b) $\pi/9$

45. (a) $7\pi/3$ (b) $-11\pi/30$

46. (a) $11\pi/6$ (b) $34\pi/15$

E3 Co-Terminal Angles

Find a positive and a negative coterminal angle for each given angle.

1) $-345°$

2) $-180°$

3) $165°$

4) $\dfrac{41\pi}{12}$

5) $\dfrac{5\pi}{12}$

6) $\dfrac{13\pi}{12}$

Source: Esther Song, Niles West High School

Answers to E3 Co-Terminal Angles (ID: 1)

1) $15°$ and $-705°$

2) $180°$ and $-540°$

3) $525°$ and $-195°$

4) $\dfrac{17\pi}{12}$ and $-\dfrac{7\pi}{12}$

5) $\dfrac{29\pi}{12}$ and $-\dfrac{19\pi}{12}$

6) $\dfrac{37\pi}{12}$ and $-\dfrac{11\pi}{12}$

Source: Esther Song, Niles West High School

D3. Rotations and Co-terminal Angles

Match the radian measure with the corresponding verbal translation

1. $\dfrac{7\pi}{3}$

2. $\dfrac{7\pi}{6}$

3. $\dfrac{2\pi}{3}$

4. $\dfrac{8\pi}{3}$

5. $\dfrac{9\pi}{4}$

6. $\dfrac{10\pi}{3}$

> A.   1 copy more than 3 rotations
>
> B.   1 copy less than 3 rotations
>
> C.   1 copy more than 2 rotations
>
> D.   1 copy less than 2 rotations
>
> E.   1 copy more than ½ of a rotation
>
> F.   1 copy less than ½ of a rotation

Source: Esther Song, Niles West High School

Name: _____

Adding Fractions
with the Unlike Denominator, Requires Simplifying

$$\frac{1}{3}$$
$$+\frac{1}{6}$$

$$\frac{1}{3}=\frac{2}{6}$$
$$+\frac{1}{6}=\frac{1}{6}$$

$$\frac{1}{3}=\frac{2}{6}$$
$$+\frac{1}{6}=\frac{1}{6}\quad \text{same}$$
$$\frac{}{6}$$

$$\frac{1}{3}=\frac{2}{6}\quad +$$
$$+\frac{1}{6}=\frac{1}{6}$$
$$\frac{3}{6}$$

$$\frac{1}{3}$$
$$+\frac{1}{6}$$
$$\frac{3}{6}=\frac{1}{2}$$

Add the fractions and simplify the answers.

a. $\frac{2}{12}$
   $+\frac{4}{6}$

b. $\frac{4}{8}$
   $+\frac{1}{4}$

c. $\frac{3}{5}$
   $+\frac{2}{10}$

d. $\frac{1}{3}$
   $+\frac{3}{9}$

e. $\frac{2}{10}$
   $+\frac{2}{5}$

f. $\frac{3}{6}$
   $+\frac{2}{12}$

g. $\frac{1}{2}$
   $+\frac{1}{10}$

h. $\frac{1}{6}$
   $+\frac{1}{3}$

i. $\frac{1}{6}$
   $+\frac{4}{12}$

j. $\frac{1}{4}$
   $+\frac{2}{8}$

k. $\frac{1}{5}$
   $+\frac{2}{10}$

l. $\frac{4}{14}$
   $+\frac{1}{7}$

m. $\frac{1}{4}$
    $\frac{1}{3}$
    $+\frac{3}{12}$

n. $\frac{1}{2}$
    $\frac{1}{10}$
    $+\frac{1}{5}$

o. $\frac{1}{14}$
    $\frac{2}{7}$
    $+\frac{1}{7}$

p. $\frac{1}{8}$
    $\frac{1}{2}$
    $+\frac{1}{8}$

Source: Esther Song, Niles West High School

E4. Calculate Supplementary and Complementary Angles

Find the supplement of each angle

1) 50 degrees   2) $\frac{5\pi}{7}$ rad 3) 18 degrees   4) 91 degrees   5) $\frac{\pi}{6}$ rad

_____   _____   _____   _____   _____

Find the complement of each angle

1) 12 degrees   2) $\frac{2\pi}{7}$ rad 3) $\frac{\pi}{3}$ rad 4) $\frac{\pi}{6}$ rad 5) 89 degrees

_____   _____   _____   _____   _____

E4. Calculate Supplementary and Complementary Angles

Find the supplement of each angle

2) 50 degrees   2) $\frac{5\pi}{7}$ rad 3) 18 degrees   4) 91 degrees   5) $\frac{\pi}{6}$ rad

_____   _____   _____   _____   _____

Find the complement of each angle

2) 12 degrees   2) $\frac{2\pi}{7}$ rad 3) $\frac{\pi}{3}$ rad 4) $\frac{\pi}{6}$ rad 5) 89 degrees

_____   _____   _____   _____   _____

Source: Esther Song, Niles West High School

A.5 conversions

NOTE   Note that when no units of angle measure are specified, *radian measure is implied*. For instance, if you write $\theta = \pi$ or $\theta = 2$, you should mean $\theta = \pi$ radians or $\theta = 2$ radians.   ▪▪

CONVERSIONS BETWEEN DEGREES AND RADIANS

1. To convert degrees to radians, multiply degrees by $\dfrac{\pi \text{ rad}}{180°}$.

2. To convert radians to degrees, multiply radians by $\dfrac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the relationship π rad $= 180°$.

■ **EXAMPLE 3   Converting from Degrees to Radians**

a. $135° = (135 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right) = \dfrac{3\pi}{4} \text{ rad}$ Multiply by $\pi/180$.

b. $540° = (540 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right) = 3\pi \text{ rad}$ Multiply by $\pi/180$.

c. $-270° = (-270 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right) = -\dfrac{3\pi}{2} \text{ rad}$ Multiply by $\pi/180$.

NOTE   If you have a calculator with a "radian-to-degree" conversion key, try using it to verify the result shown in part (c) of Example 4.   ▪▪

■ **EXAMPLE 4   Converting from Radians to Degrees**

a. $-\dfrac{\pi}{2} \text{ rad} = \left(-\dfrac{\pi}{2} \text{ rad}\right)\left(\dfrac{180 \text{ deg}}{\pi \text{ rad}}\right) = -90°$ Multiply by $180/\pi$.

b. $\dfrac{9\pi}{2} \text{ rad} = \left(\dfrac{9\pi}{2} \text{ rad}\right)\left(\dfrac{180 \text{ deg}}{\pi \text{ rad}}\right) = 810°$ Multiply by $180/\pi$.

c. $2 \text{ rad} = (2 \text{ rad})\left(\dfrac{180 \text{ deg}}{\pi \text{ rad}}\right) = \dfrac{360}{\pi} \approx 114.59°$ Multiply by $180/\pi$.

TECHNOLOGY

$1' = 1 \text{ minute} = \frac{1}{60}(1°)$

$1'' = 1 \text{ second} = \frac{1}{3600}(1°)$.

With calculators it is convenient to use *decimal* degrees to denote fractional parts of degrees. Historically, however, fractional parts of degrees were expressed in *minutes* and *seconds*, using the prime (′) and double prime (″) notations, respectively. Consequently, an angle of 64 degrees, 32 minutes, and 47 seconds, is represented by $\theta = 64° \ 32' \ 47''$. Many calculators have special keys for converting an angle in degrees, minutes, and seconds (D° M′ S″) into decimal degree form, and vice versa.

In Exercises 47–54, convert the measure from degrees to radians. Round to three decimal places.

47. 115°

48. 87.4°

49. −216.35°

50. −48.27°

51. 532°

52. 0.54°

53. −0.83°

54. 345°

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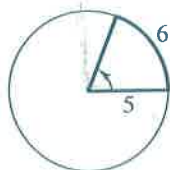**68.** (a) −345.12°   (b) 0.45

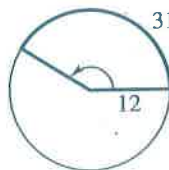**69.** (a) 2.5   (b) −3.58

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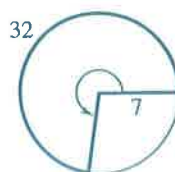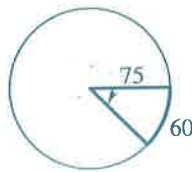In Exercises 71–74, find the angle in radians.

71.

72.

73.

74.

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81.	6 meters	2 radians
82.	40 centimeters	$3\pi/4$ radians

Distance Between Cities   In Exercises 83–86, find the distance between the cities. Assume that earth is a sphere of radius 4000 miles and the cities are on the same meridian (one city is due north of the other).

	City	Latitude
83.	Dallas, Texas	32° 47′ 9″N
	Omaha, Nebraska	41° 15′ 42″N
84.	San Francisco, California	37° 46′ 39″N
	Seattle, Washington	47° 36′ 32″N
85.	Miami, Florida	25° 46′ 37″N
	Erie, Pennsylvania	42° 7′ 15″N
86.	Johannesburg, South Africa	26° 10′ S
	Jerusalem, Israel	31° 47′ N

87. *Difference in Latitudes*   Assuming that earth is a sphere of radius 6378 kilometers, what is the difference in latitude of two cities, one of which is 600 kilometers due north of the other?

88. *Difference in Latitudes*   Assuming that earth is a sphere of radius 6378 kilometers, what is the difference in latitude of two cities, one of which is 800 kilometers due north of the other?

ID: 1

Name_____

E5 Converting Radian & Degrees

Date_____ Period____

Convert each radian measure into degrees. *(choose 2)*

1) $\dfrac{5\pi}{6}$

2) $\dfrac{7\pi}{4}$

3) $\dfrac{2\pi}{3}$

4) $\dfrac{3\pi}{4}$

Convert each degree measure into radians. *(choose 2)*

5) 240°

6) 60°

7) 300°

8) 330°

Convert each degree measure into radians and each radian measure into degrees. *(choose 3)*

9) $-\dfrac{13\pi}{6}$

10) -525°

11) $\dfrac{11\pi}{6}$

12) 345°

13) $-\dfrac{17\pi}{6}$

14) 930°

Source: Esther Song, Niles West High School

Answers to E5 Converting Radian & Degrees (ID: 1)

1) 150°

2) 315°

3) 120°

4) 135°

5) $\dfrac{4\pi}{3}$

6) $\dfrac{\pi}{3}$

7) $\dfrac{5\pi}{3}$

8) $\dfrac{11\pi}{6}$

9) -390°

10) $-\dfrac{35\pi}{12}$

11) 330°

12) $\dfrac{23\pi}{12}$

13) -510°

14) $\dfrac{31\pi}{6}$